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The AC magnetoresistance in inhomogeneous solids

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Abstract. The theory of AC magnetoresistance in inhomogeneous solids is developed using the effective-medium theory. Calculations are performed for systems which consist of random high- and low-conductivity regions. The weak magnetic field $\boldsymbol{H}$ is directed along the z axis. The effective conductivity changes $\Delta\sigma_{\rm m}^{xx}$ and $\Delta\sigma_{\rm m}^{zz}$ and the magnetoresistances $\Delta\rho_{\rm m}^{xx}$ and $\Delta\rho_{\rm m}^{zz}$ in both the low- and the high-frequency regions are calculated up to the H^2 approximation. The calculations show that, owing to inhomogeneities, plateaux of the finite values $\Delta\sigma_{\rm m}^{zz}$ and $\Delta\rho_{\rm m}^{zz}$ in the low-frequency region occur. It is possible to have both $\Delta\rho_{\rm m}^{xx} > \Delta\rho_{\rm m}^{zz}$ and $\Delta\rho_{\rm m}^{xx} < \Delta\rho_{\rm m}^{zz}$. In the high-frequency limit the equalities $\Delta\sigma_{\rm m}^{zz} = \Delta\rho_{\rm m}^{zz} = 0$ hold as in homogeneous solids.

1. Introduction

Theoretical investigations of the AC kinetic phenomena in inhomogeneous solids have been reported in many papers. Calculations of the AC conductivity have been performed, for example, by Springett (1973), Webman *et al* (1977b) and Sinkkonen (1981). They have shown that the effective conductivity has strong dispersion at the frequency $\omega_0 \sim \tau_{\rm M}^{-1}$ and has low- and high-frequency plateaux. Here $\tau_{\rm M}$ is the Maxwell relaxation time. In the studies by Fishchuk (1983, 1986) a similar result was obtained for the AC effective Hall conductivity and the effective Hall mobility. These results were used by Jaouen *et al* (1986) to interpret the experimental data on the Hall mobility in silicon with arsenic ion implantation.

The AC magnetoresistance in semiconductors with random dielectric inclusions and in highly inhomogeneous semiconductors was investigated by Fishchuk (1987, 1989). In the low-frequency region the longitudinal magnetoresistance appears as a result of inclusions. This kind of magnetoresistance is absent in the high-frequency region and in homogeneous solids. In the present paper we develop the general theory of AC magnetoresistance in inhomogeneous solids in the presence of a weak magnetic field. Calculations are performed for semiconductors with random low-conductivity inclusions. As in the cited papers we use the effective-medium theory (EMT). The validity of the EMT results was corroborated by comparison with numerical simulation data in the presence of both magnetic (Webman *et al* 1977a) and alternating electrical (Webman *et al* 1977b) fields.

2. Theory

We consider a semiconductor with random macroscopic fluctuations of the electrostatic potential. The average fluctuation space size is much larger than the electron

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mean free path. In this case the local values of kinetic coefficients may be introduced at the point $\boldsymbol{r}$. We suppose that the applied external magnetic field $\boldsymbol{H}$ is directed along the z axis. Let us apply the external electrical field $\boldsymbol{E}(t) = \boldsymbol{E}_0 \exp(\mathrm{i}\omega t)$ of frequency ω. Then at the point $\boldsymbol{r}$ we have the electrical field $\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0(\boldsymbol{r}) \exp(\mathrm{i}\omega t)$, where $\langle \boldsymbol{E}_0(\boldsymbol{r}) \rangle = \boldsymbol{E}_0$. The angular brackets denote the space averaging. We investigate the frequency region, where $\omega < \tau^{-1}$. Here τ is the electron mean free time. When the displacement current is taken into account, the complex conductivity tensor $\hat{\sigma}^*(\boldsymbol{r})$ has the form $\hat{\sigma}^*(\boldsymbol{r}) = \hat{\sigma}(\boldsymbol{r}) + \mathrm{i}\omega\hat{\epsilon}_0/4\pi$. Here we suppose that the dielectric permeability $\hat{\epsilon}_0$ is constant over the whole volume of the system. We introduce the effective conductivity tensor $\hat{\sigma}^*_{\mathrm{m}}$ by

$$\langle \boldsymbol{J}(\boldsymbol{r}) \rangle = \langle \hat{\sigma}^*(\boldsymbol{r}) \times \boldsymbol{E}(\boldsymbol{r}) \rangle = \hat{\sigma}^*_{\mathrm{m}} \times \langle \boldsymbol{E}_0(\boldsymbol{r}) \rangle = \hat{\sigma}^*_{\mathrm{m}} \times \boldsymbol{E}_0 \tag{1}$$

where $\boldsymbol{J}(\boldsymbol{r})$ is the local current density. The value $\hat{\sigma}^*_{\mathrm{m}}$ has the form $\hat{\sigma}^*_{\mathrm{m}} = \hat{\sigma}_{\mathrm{m}} + \mathrm{i}\omega\hat{\epsilon}_{\mathrm{m}}/4\pi$, where $\hat{\epsilon}_{\mathrm{m}}$ is the effective dielectric permeability of the system. Further we write $\hat{\sigma}^*_{\mathrm{m}} = \hat{\sigma}_{\mathrm{m}} + \mathrm{i}\omega\epsilon_0\hat{I}/4\pi$. Here $\hat{\sigma}_{\mathrm{m}}$ is the complex value and $\hat{\epsilon}_{\mathrm{m}} = \epsilon_0\hat{I} + 4\pi\,\mathrm{Im}(\hat{\sigma}_{\mathrm{m}}/\omega)$. However, in the low- and high-frequency limits considered, we have $\mathrm{Im}\,\hat{\sigma}_{\mathrm{m}} \ll \mathrm{Re}\,\hat{\sigma}_{\mathrm{m}}$, i.e. $\hat{\sigma}_{\mathrm{m}} \simeq \mathrm{Re}\,\hat{\sigma}_{\mathrm{m}}$. Let us write $\hat{\sigma}^*(\boldsymbol{r})$ as

$$\hat{\sigma}^*(\boldsymbol{r}) = \hat{\sigma}^*_{\mathrm{m}} + \Delta\hat{\sigma}(\boldsymbol{r}) \qquad \Delta\hat{\sigma}(\boldsymbol{r}) = \hat{\sigma}(\boldsymbol{r}) - \hat{\sigma}_{\mathrm{m}}. \tag{2}$$

Inserting (2) into (1) we obtain

$$\langle \boldsymbol{J}(\boldsymbol{r}) \rangle = \hat{\sigma}^*_{\mathrm{m}} \times \boldsymbol{E}_0 + \langle \Delta\hat{\sigma}(\boldsymbol{r}) \times \boldsymbol{E}_0(\boldsymbol{r}) \rangle. \tag{3}$$

We see that it is necessary to calculate the value $\Delta\hat{\sigma}(\boldsymbol{r}) \times \boldsymbol{E}_0(\boldsymbol{r})$. Then from the condition $\langle \Delta\hat{\sigma}(\boldsymbol{r}) \times \boldsymbol{E}_0(\boldsymbol{r}) \rangle = 0$ we can find $\hat{\sigma}_{\mathrm{m}}$ and consequently the value $\hat{\sigma}^*_{\mathrm{m}}$.

Stroud (1975) obtained the equations required to calculate the DC effective conductivity tensor in inhomogeneous systems in the presence of a magnetic field using the EMT. We extend the theory of Stroud to calculate the AC conductivity tensor $\hat{\sigma}_{\mathrm{m}}$. We obtain the following equation:

$$\langle (\hat{I} - \Delta\hat{\sigma} \cdot \hat{\Gamma})^{-1} \Delta\hat{\sigma} \rangle = 0. \tag{4}$$

From here we find

$$\left\langle [(1 - \Delta\sigma_{xx}\Gamma_{xx})\Delta\sigma_{xx} - \Gamma_{xx}(\Delta\sigma_{yx})^2]/[(1 - \Delta\sigma_{xx}\Gamma_{xx})^2 + \Gamma^2_{xx}(\Delta\sigma_{yx})^2] \right\rangle = 0 \tag{5}$$

$$\langle \Delta\sigma_{zz}/(1 - \Delta\sigma_{zz}\Gamma_{zz}) \rangle = 0 \tag{6}$$

$$\left\langle \Delta\sigma_{yx}/[(1 - \Delta\sigma_{xx}\Gamma_{xx})^2 + \Gamma^2_{xx}(\Delta\sigma_{yx})^2] \right\rangle = 0 \tag{7}$$

$$\Gamma_{xx} = \Gamma_{yy} = \tfrac{1}{2}\{1/\epsilon[\sigma^{zz}_{\mathrm{m}} + \mathrm{i}(\omega/4\pi)\epsilon_0]\}\left\{1 - [(1-\epsilon)^{-1/2}\sin^{-1}(\sqrt{\epsilon})]/\sqrt{\epsilon}\right\} \tag{8}$$

$$\Gamma_{zz} = \{1/\epsilon[\sigma^{zz}_{\mathrm{m}} + \mathrm{i}(\omega/4\pi)\epsilon_0]\}\left\{1 - [(1-\epsilon)^{1/2}\sin^{-1}(\sqrt{\epsilon})]/\sqrt{\epsilon}\right\} \tag{9}$$

$$\epsilon = (\sigma^{zz}_{\mathrm{m}} - \sigma^{xx}_{\mathrm{m}})/[\sigma^{zz}_{\mathrm{m}} + \mathrm{i}(\omega/4\pi)\epsilon_0]. \tag{10}$$

As we see, the frequency ω is only in $\hat{\Gamma}$.

3. Magnetoresistance at weak magnetic fields

The components of the tensor $\hat{\sigma}(r)$ in inhomogeneous systems have the same form as in homogeneous systems. Consequently for weak magnetic fields we can write

$$\sigma_{xx} = \sigma - \Delta\sigma'_{xx} \qquad \sigma_{yy} = \sigma_{xx} \qquad \sigma_{zz} = \sigma \tag{11}$$

$$\sigma_{yx} = \sigma a_{21}(\mu H/c) \qquad \Delta\sigma'_{xx} = \sigma a_{11}(\mu H/c)^2 \tag{12}$$

$$\mu = (e/m)\langle\tau\rangle \qquad a_{11} = \frac{\langle\tau^3\rangle}{\langle\tau\rangle^3} \qquad a_{21} = \langle\tau^2\rangle/\langle\tau\rangle^2. \tag{13}$$

The angular brackets in (13) denote the energy averaging.

As components of effective tensor $\hat{\sigma}_{\mathrm{m}}$ we choose

$$\sigma_{\mathrm{m}}^{xx} = \sigma_{\mathrm{m}}^0 - \Delta\sigma_{\mathrm{m}}^{xx} \qquad \sigma_{\mathrm{m}}^{yy} = \sigma_{\mathrm{m}}^{xx} \qquad \sigma_{\mathrm{m}}^{zz} = \sigma_{\mathrm{m}}^0 - \Delta\sigma_{\mathrm{m}}^{zz} \tag{14}$$

$$\sigma_{\mathrm{m}}^{yx} = \sigma_{21}a_{21}(\mu H/c) \qquad \Delta\sigma_{\mathrm{m}}^{xx} = \sigma_{11}a_{11}(\mu H/c)^2 \qquad \Delta\sigma_{\mathrm{m}}^{zz} = \sigma_{33}a_{11}(\mu H/c)^2. \tag{15}$$

The values σ_{11}, σ_{33}, σ_{21} and σ_{m}^0 depend on the degree of inhomogeneity and must be calculated. For this purpose we insert (11)–(15) into (5)–(7) and expand the results in powers of H up to the quadratic averaged term. Every term of the series must be equated to zero. We obtain the following system of equations:

$$(\sigma_{11} - \sigma_{\mathrm{m}}^0)\langle A\rangle + \langle(2\sigma_{11} + \sigma)(\sigma - \sigma_{\mathrm{m}}^0)A^2\rangle + \tfrac{1}{5}[(\sigma_{11} - \sigma_{33})/B]\langle(\sigma - \sigma_{\mathrm{m}}^0)^2A^2\rangle + 3KB\langle(\sigma - \sigma_{21})^2A^3\rangle = 0 \tag{16}$$

$$\sigma_{33} = \sigma_{11}\left\{2\langle(\sigma - \sigma_{\mathrm{m}}^0)^2A^2\rangle/[15B^2\langle A^2\rangle - 3\langle(\sigma - \sigma_{\mathrm{m}}^0)A^2\rangle\right\} \tag{17}$$

$$\langle(\sigma - \sigma_{21})A^2\rangle = 0 \qquad \langle(\sigma - \sigma_{\mathrm{m}}^0)A\rangle = 0 \tag{18}$$

where

$$A = 1/[\sigma + 2\sigma_{\mathrm{m}}^0 + \mathrm{i}3\omega(\epsilon_0/4\pi)] \qquad B = \sigma_{\mathrm{m}}^0 + \mathrm{i}\omega(\epsilon_0/4\pi) \qquad K = a_{21}^2/a_{11}. \tag{19}$$

For further calculations we must choose the distribution function of the value σ. We study a semiconductor with random low-conductivity inclusions. We suppose that p and $1 - p$ are the parts of the system volume with the conductivities σ_0 and σ_1, respectively ($\sigma_1/\sigma_0 \equiv X_1 < 1$).

4. Frequency dependence of the magnetoresistance

We consider both low-frequency regions, where $\omega \ll 4\pi|\sigma_{\mathrm{m}}^{zz}|/\epsilon_0$, and high-frequency regions, where $\tau^{-1} > \omega \gg 4\pi|\sigma_{\mathrm{m}}^{zz}|/\epsilon_0$.

Let us examine the low-frequency region. In the limiting case we can take $\omega = 0$. In this case we perform averaging in (16)–(18) and obtain

$$\sigma_{11}/\sigma_{\rm m}^0 = (D_1 - KD_2)/(D_1 + D_3) \tag{20}$$

$$\sigma_{33}/\sigma_{\rm m}^0 = (\sigma_{11}/\sigma_{\rm m}^0)\tfrac{2}{3}[(1-X_{\rm m}^0)^2 a_0 + (X_1 - X_{\rm m}^0)^2 a_1] \times \left\{[4(X_{\rm m}^0)^2 - 1 + 2X_{\rm m}^0]a_0 + [4(X_{\rm m}^0)^2 - X_1^2 + 2X_1X_{\rm m}^0]a_1\right\}^{-1} \tag{21}$$

$$\sigma_{21}/\sigma_{\rm m}^0 = (a_0 + a_1X_1)/(a_0+a_1)X_{\rm m}^0 \tag{22}$$

$$\sigma_{\rm m}^0/\sigma_0 = a + (a^2 + \tfrac{1}{2}X_1)^{1/2} \tag{23}$$

where

$$D_1 = p/(1+2X_{\rm m}^0)^2 + (1-p)/(X_1 + 2X_{\rm m}^0)^2 \tag{24}$$

$$D_2 = p(1 - X_{\rm m}^{21})^2/(1+2X_{\rm m}^0)^3 + (1-p)(X_1 - X_{\rm m}^{21})^2/(X_1+2X_{\rm m}^0)^3 \tag{25}$$

$$D_3 = [(1-\sigma_{33}/\sigma_{11})/15X_{\rm m}^0][p(1-X_{\rm m}^0)^2/(1+2X_{\rm m}^0)^2 + (1-p)(X_1 - X_{\rm m}^0)^2/(X_1+2X_{\rm m}^0)^2] \tag{26}$$

$$a_0 = p(X_1 + 2X_{\rm m}^0)^2 \qquad a_1 = (1-p)(1+2X_{\rm m}^0)^2 \tag{27}$$

$$a = \tfrac{1}{2}[\tfrac{1}{2}(3p-1)(1-X_1) + \tfrac{1}{2}X_1] \tag{28}$$

$$X_{\rm m}^0 = \sigma_{\rm m}^0/\sigma_0 \qquad X_{\rm m}^{21} = \sigma_{21}/\sigma_0. \tag{29}$$

If $p \to 1$, we have $\sigma_{11}/\sigma_{\rm m}^0 \to \sigma_{21}/\sigma_{\rm m}^0 \to \sigma_{\rm m}^0/\sigma_0 \to 1$, $\sigma_{33}/\sigma_{\rm m}^0 \to 0$. If $p \to 0$, we obtain $\sigma_{11}/\sigma_{\rm m}^0 \to \sigma_{21}/\sigma_{\rm m}^0 \to 1$, $\sigma_{33}/\sigma_{\rm m}^0 \to 0$, $\sigma_{\rm m}^0/\sigma_0 \to \sigma_1/\sigma_0$.

Let us consider the case of dielectric inclusions ($\sigma_1 \to 0$). Then from (20)–(23) for $1 \geqslant p > p_{\rm c}$ ($p_{\rm c}$ is the percolation threshold) one easily obtains

$$\sigma_{11}/\sigma_{\rm m}^0 = \varphi_1 + K\varphi_2 \qquad \sigma_{33}/\sigma_{\rm m}^0 = (\sigma_{11}/\sigma_{\rm m}^0)\tfrac{3}{2}(1-p)/(6p-1) \tag{30}$$

$$\sigma_{21}/\sigma_{\rm m}^0 = 2(3p-1)/(3p+1) \qquad \sigma_{\rm m}^0/\sigma_0 = (3p-1)/2. \tag{31}$$

Here

$$\varphi_1 = 4(6p-1)/(21p-1) \qquad \varphi_2 = -72p(6p-1)(1-p)/(3p+1)^2(21p-1). \tag{32}$$

If $p \to p_{\rm c}$ we have $\sigma_{11}/\sigma_{\rm m}^0 \to \sigma_{33}/\sigma_{\rm m}^0 \to 2(1-K)/3$, $\sigma_{21}/\sigma_{\rm m}^0 \to \sigma_{\rm m}^0/\sigma_0 \to 0$.

In figure 1 the functions $\sigma_{\rm m}^0/\sigma_0$, $[(\mu H/c)a_{21}]^{-1}\sigma_{\rm m}^{yx}/\sigma_{\rm m}^0 = \sigma_{21}/\sigma_{\rm m}^0$, $\Delta\sigma_{\rm m}^{zz}/\Delta\sigma_{\rm m}^{xx} = \sigma_{33}/\sigma_{11}$ versus p obtained from (21)–(23) are shown. The broken curves here and below are derived for the case $\sigma_1 = 0$.

In figures 2 and 3 the functions $[(\mu H/c)^2 a_{11}]^{-1}\Delta\sigma_{\rm m}^{xx}/\sigma_{\rm m}^0 = \sigma_{11}/\sigma_{\rm m}^0$ and $[(\mu H/c)^2 a_{11}]^{-1}\Delta\sigma_{\rm m}^{zz}/\sigma_{\rm m}^0 = \sigma_{33}/\sigma_{\rm m}^0$ versus p obtained from (20) and (21) for different K-values are shown.

Now we consider the transverse and longitudinal magnetoresistances $\Delta\rho_{\rm m}^{\alpha\alpha} = \rho_{\rm m}^{\alpha\alpha} - \rho_{\rm m}^0$ using the expressions

$$\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^0 = \Delta\sigma_{\rm m}^{xx}/\sigma_{\rm m}^0 - (\sigma_{\rm m}^{yx}/\sigma_{\rm m}^0)^2 \qquad \Delta\rho_{\rm m}^{zz}/\rho_{\rm m}^0 = \Delta\sigma_{\rm m}^{zz}/\sigma_{\rm m}^0 \qquad \rho_{\rm m}^0 = 1/\sigma_{\rm m}^0. \tag{33}$$

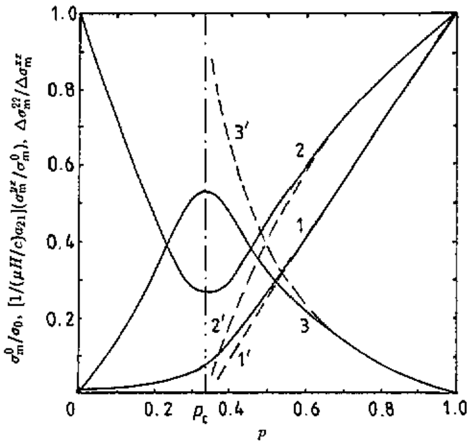

Figure 1. Dependences of σ_m^0/σ_0 (curve 1), σ_m^{yx}/σ_m^0 (curve 2) and $\Delta\sigma_m^{zz}/\Delta\sigma_m^{xx}$ (curve 3) on p for $\sigma_1/\sigma_0 = 0.01$. The broken curves here and below correspond to the case when $\sigma_1 = 0$.

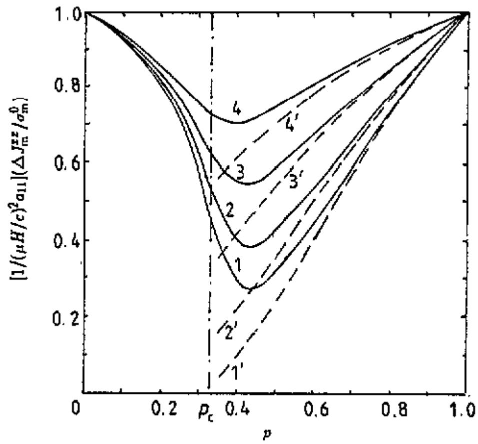

Figure 2. Dependences of $\Delta\sigma_m^{xx}/\sigma_m^0$ on p for $\sigma_1/\sigma_0 = 0.01$ at various K: curve 1, $K = 1.0$; curve 2, $K = 0.8$; curve 3, $K = 0.5$; curve 4, $K = 0.2$.

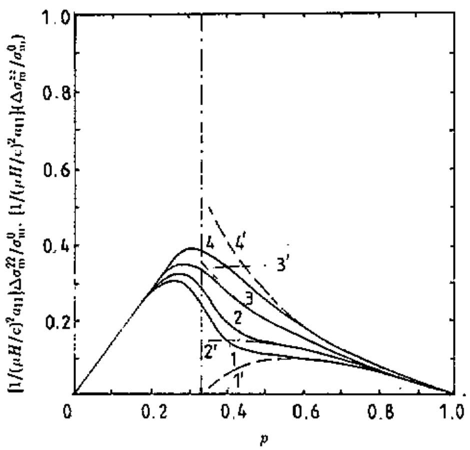

Figure 3. Dependences of $\Delta\sigma_m^{zz}/\sigma_m^0$ and $\Delta\rho_m^{zz}/\rho_m^0$ on p for $\sigma_1/\sigma_0 = 0.01$ at various K: curve 1, $K = 1.0$; curve 2, $K = 0.8$; curve 3, $K = 0.5$; curve 4, $K = 0.2$.

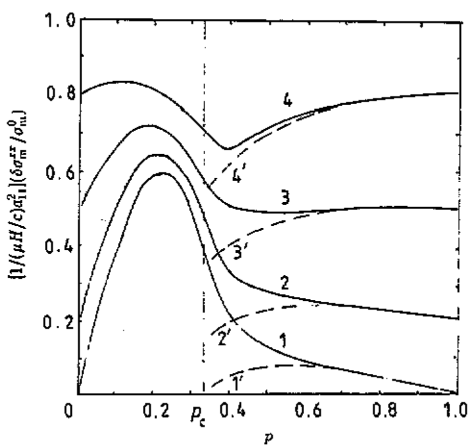

Figure 4. Dependence of $\Delta\rho_m^{xx}/\rho_m^0$ on p for $\sigma_1/\sigma_0 = 0.01$ at various K: curve 1, $K = 1.0$; curve 2, $K = 0.8$; curve 3, $K = 0.5$; curve 4, $K = 0.2$.

In our case the values $\Delta\sigma_m^{\alpha\alpha}$ and $\Delta\rho_m^{\alpha\alpha}$ are positive. Inserting (15) into (33) we obtain

$$\begin{aligned}\Delta\rho_m^{xx}/\rho_m^0 &= [\sigma_{11}/\sigma_m^0 - (\sigma_{21}/\sigma_m^0)^2 K]a_{11}(\mu H/c)^2 \\ \Delta\rho_m^{zz}/\rho_m^0 &= (\sigma_{33}/\sigma_m^0)a_{11}(\mu H/c)^2.\end{aligned} \tag{34}$$

In the case of dielectric inclusions using (30) and (31) we find that

$$\Delta\rho_m^{xx}/\rho_m^0 = [4(6p-1)/(21p-1)]\{1 - K[(3p-1)^2(21p-1)$$

$$+ 18p(1-p)(6p-1)]/(6p-1)(3p+1)^2\}a_{11}(\mu H/c)^2 \tag{35}$$

$$\Delta\rho_{\mathrm{m}}^{zz}/\rho_{\mathrm{m}}^{0} = [6(1-p)/(21p-1)]\left\{1 - K[18p(1-p)/(3p+1)^2]\right\} a_{11}(\mu H/c)^2. \tag{36}$$

If $K = 1$ (dispersion of τ is absent), equations (35) and (36) become

$$\Delta\rho_{\mathrm{m}}^{xx}/\rho_{\mathrm{m}}^{0} = [36p(1-p)(3p-1)/(21p-1)(3p+1)^2](\mu H/c)^2 \tag{37}$$

$$\Delta\rho_{\mathrm{m}}^{xx}/\rho_{\mathrm{m}}^{0} = [6(1-p)(3p-1)(9p-1)/(21p-1)(3p+1)^2](\mu H/c)^2. \tag{38}$$

In the limiting case $q = 1 - p \ll 1$ we have

$$\Delta\rho_{\mathrm{m}}^{xx}/\rho_{\mathrm{m}}^{0} = \tfrac{9}{40}q(\mu H/c)^2 \qquad \Delta\rho_{\mathrm{m}}^{zz}/\rho_{\mathrm{m}}^{0} = \tfrac{3}{10}q(\mu H/c)^2. \tag{39}$$

The last results coincide with the formulae in the paper by Pohoryles and Figielski (1975).

In figure 4 the functions $[(\mu H/c)^2 a_{11}]^{-1}\Delta\rho_{\mathrm{m}}^{xx}/\rho_{\mathrm{m}}^{0}$ versus p obtained from (34) using (20) and (22) for different K-values are shown. As we can conclude from (33) the functions $[(\mu H/c)^2 a_{11}]^{-1}\Delta\rho_{\mathrm{m}}^{zz}/\rho_{\mathrm{m}}^{0}$ versus p coincide with the functions $[(\mu H/c)^2 a_{11}]^{-1}\Delta\sigma_{\mathrm{m}}^{zz}/\sigma_{\mathrm{m}}^{0}$ versus p in figure 3.

We see from figures 1–4 that there are distinctions between the full and broken curves in the vicinity of p_c as in the case of the DC conductivity (Kirkpatrick 1973).

Let us study the correlation of transverse and longitudinal magnetoresistances. For this purpose we use the equality $\Delta\rho_{\mathrm{m}}^{xx} = \Delta\rho_{\mathrm{m}}^{zz}$. Taking into account (35) and (36) we find that

$$K = \tfrac{5}{2}(3p+1)^2/[(3p-1)(21p-1) + 45p(1-p)]. \tag{40}$$

In figure 5 the dependence of K versus p at $p_c < p \leqslant 1$ is shown.

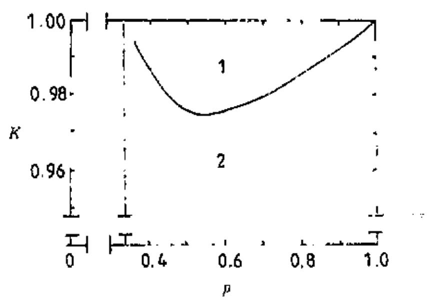

Figure 5. Dependence of K on p for $\sigma_1 = 0$ and the condition $\Delta\rho_{\mathrm{m}}^{zz} = \Delta\rho_{\mathrm{m}}^{xx}$ ($\Delta\rho_{\mathrm{m}}^{zz} > \Delta\rho_{\mathrm{m}}^{xx}$ in area 1, and $\Delta\rho_{\mathrm{m}}^{zz} < \Delta\rho_{\mathrm{m}}^{xx}$ in area 2).

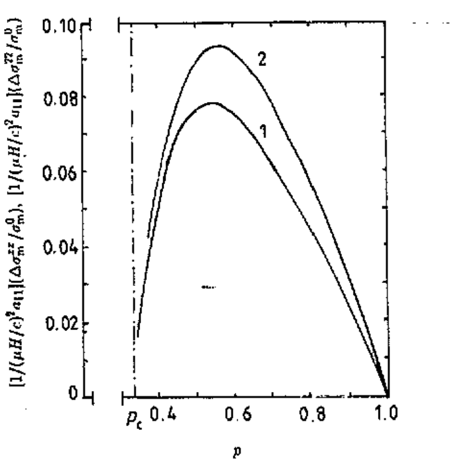

Figure 6. Dependences of $\Delta\rho_{\mathrm{m}}^{xx}/\rho_{\mathrm{m}}^{0}$ (curve 1) and $\Delta\rho_{\mathrm{m}}^{zz}/\rho_{\mathrm{m}}^{0}$ (curve 2) on p for $\sigma_1 = 0$ and $K = 1.0$.

It is easy to see from (35) and (36) that in regions 1 and 2 the inequalities $\Delta\rho_{\rm m}^{zz} > \Delta\rho_{\rm m}^{xx}$ and $\Delta\rho_{\rm m}^{zz} < \Delta\rho_{\rm m}^{xx}$, respectively, occur. In figure 6 the functions $[(\mu H/c)^2 a_{11}]^{-1}\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^0$ and $[(\mu H/c)^2 a_{11}]^{-1}\Delta\rho_{\rm m}^{zz}/\rho_{\rm m}^0$ versus p obtained from (37) and (38) for $K = 1$ are shown. These curves coincide with the broken curves 1′ in figures 3 and 4. For the values $p > p_{\rm c}$ we have the ratio $\Delta\rho_{\rm m}^{zz}/\Delta\rho_{\rm m}^{xx} = (9p-1)/6p < 1$.

Next we consider the high-frequency region. In this region, only the first term in equation (16) has a finite value. Consequently we have $\sigma_{11} = \sigma_{\rm m}^0$. From (17) we obtain $\sigma_{33} = 0$. From (18) we find that $\sigma_{\rm m}^0 = \langle\sigma\rangle$. Then we write

$$
\begin{aligned}
&[1/(\mu H/c)^2 a_{11}](\Delta\sigma_{\rm m}^{xx}/\sigma_{\rm m}^0) = [1/(\mu H/c) a_{21}](\sigma_{\rm m}^{yx}/\sigma_{\rm m}^0) = 1 \\
&[1/(\mu H/c)^2 a_{11}](\Delta\sigma_{\rm m}^{zz}/\sigma_{\rm m}^0) = 0
\end{aligned}
\tag{41}
$$

where

$$\sigma_{\rm m}^0/\sigma_0 = p + (1-p)\sigma_1/\sigma_0. \tag{42}$$

For the magnetoresistance we find that

$$\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^0 = (1-K)a_{11}(\mu H/c)^2 \qquad \Delta\rho_{\rm m}^{zz}/\rho_{\rm m}^0 = 0. \tag{43}$$

Now the conditions for the low- and high-frequency regions can be rewritten as

$$\omega < \omega_{\rm l} = (4\pi/\epsilon_0)\sigma_{\rm m}^{(\rm l)} \qquad \omega > \omega_{\rm h} = (4\pi/\epsilon_0)\sigma_{\rm m}^{(\rm h)} \tag{44}$$

where $\sigma_{\rm m}^{(\rm l)}$ and $\sigma_{\rm m}^{(\rm h)}$ are the effective values of the low- and high-frequency conductivities, respectively. In the frequency band $\omega_{\rm l} < \omega < \omega_{\rm h}$ the transition from the low- to high-frequency regions occurs. The length of the intermediate-frequency band depends on p. For example, if $\sigma_1 = 0$, we have $\omega_{\rm h} - \omega_{\rm l} = 2\pi\sigma_0(1-p)/\epsilon_0$. If $p \to 1$, the intermediate-frequency band disappears naturally. If $p \to p_{\rm c}$, the intermediate-frequency band length has the maximum value $\omega_{\rm h} - \omega_{\rm l} = 4\pi\sigma_0/3\epsilon_0$.

5. Discussion and conclusions

The theory of the AC magnetoresistance in random inhomogeneous solids using the EMT is developed. We consider the case of weak magnetic fields. The frequency dependences of the changes in conductivity due to $\boldsymbol{H}$ and the magnetoresistance in both the transverse and the longitudinal directions to $\boldsymbol{H}$ are investigated. Calculations are performed for semiconductors with random low-conductivity inclusions. We investigate the changes in conductivity and magnetoresistance in both the low-frequency ($\omega < \omega_{\rm l}$) and high-frequency ($\tau^{-1} > \omega > \omega_{\rm h}$) limits. As can be easily seen, the low- and high-frequency asymptotic values obtained change very weakly in the large frequency limits, i.e. we have low- and high-frequency plateaux. In the frequency band $\omega_{\rm l} < \omega < \omega_{\rm h}$ there is strong dispersion and the transition from the low- to high-frequency plateau occurs.

The calculations show that in the low-frequency limit the curve of $\sigma_{\rm m}^{yx}/\sigma_{\rm m}^0$ versus p (figure 1) has a minimum and the ratio $\Delta\sigma_{\rm m}^{zz}/\Delta\sigma_{\rm m}^{xx} < 1$ has a maximum at the percolation threshold $p_{\rm c}$. The ratio $\Delta\sigma_{\rm m}^{zz}/\Delta\sigma_{\rm m}^{xx}$ depends on p but does not depend on K. $\Delta\sigma_{\rm m}^{zz}$ and $\Delta\sigma_{\rm m}^{xx}$ depend on both p and K (figures 2 and 3).

The curves of $\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^{0}$ versus p strongly depend on K (figure 4). As follows from (33) the curves of $\Delta\rho_{\rm m}^{zz}/\rho_{\rm m}^{0}$ versus p coincide with the curves of $\Delta\sigma_{\rm m}^{zz}/\sigma_{\rm m}^{0}$ (figure 3). The ratio $\Delta\rho_{\rm m}^{zz}/\Delta\rho_{\rm m}^{xx}$ depends on both p and K. In areas 1 and 2 in figure 5 we have $\Delta\rho_{\rm m}^{zz} > \Delta\rho_{\rm m}^{xx}$ and $\Delta\rho_{\rm m}^{zz} < \Delta\rho_{\rm m}^{xx}$, respectively. If $K = 1$ as follows from figure 6 and (37) and (38) for $1 > p > p_c$, we have $\Delta\rho_{\rm m}^{zz} > \Delta\rho_{\rm m}^{xx}$.

Let us discuss the frequency behaviour of the conductivity changes and magnetoresistance. The finite values of $\Delta\sigma_{\rm m}^{xx}/\sigma_{\rm m}^{0}$ and $\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^{0}$ in the low-frequency limit go to finite values in the high-frequency limit with increasing ω. If $K = 1$ we have that $\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^{0} \neq 0$ in the low-frequency limit, and $\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^{0} = 0$ in the high-frequency limit.

The finite values of $\Delta\sigma_{\rm m}^{zz}/\sigma_{\rm m}^{0}$ and $\Delta\rho_{\rm m}^{zz}/\rho_{\rm m}^{0}$ in the low-frequency limit decrease to zero when we go from the low- to the high-frequency limit. In homogeneous systems ($p = 1$ or $p = 0$) the values of $\Delta\sigma_{\rm m}^{xx}/\sigma_{\rm m}^{0}$ and $\Delta\rho_{\rm m}^{xx}/\rho_{\rm m}^{0}$ do not depend on the frequency, and the values of $\Delta\sigma_{\rm m}^{zz}/\sigma_{\rm m}^{0}$ and $\Delta\rho_{\rm m}^{zz}/\rho_{\rm m}^{0}$ are equal to zero for all frequencies.

The length of the transition band from the low- to the high-frequency region depends on p. At the percolation threshold p_c the length has the maximum value.

In conclusion we discuss the conditions of experimental observation of our results. We must point out that curves in all figures are calculated under the condition of fixed values of σ_0 and σ_1. This condition can occur, for example, in prepared two-component solid mixtures with various p. If we create a low-conductivity region by irradiation or ion implantation in a semiconductor, the values σ_0, σ_1 and σ_1/σ_0 can be changed because of the creation of point defects in the system. However, to obtain the experimental frequency dependences of magnetoresistance there are no significant problems. Jaouen *et al* (1986) and Christofidés *et al* (1989) obtained low- and high-frequency plateaux in the Hall mobility for silicon with ion implantation. We can expect to obtain similar frequency plateaux in the magnetoresistance in inhomogeneous solids.

References

Christofidés C, Jaouen H and Chibaudo G 1989 *J. Appl. Phys.* **65** 4832–9
Fishchuk I I 1983 *Fiz. Tekh. Poluprov.* **17** 1189–94
—— 1986 *Phys. Status Solidi* a **93** 675–84
—— 1987 *Fiz. Tverd. Tela* **29** 2705–9
—— 1989 *Fiz. Tverd. Tela* **31** 135–9
Jaouen H, Chibaudo G and Christofidés C 1986 *J. Appl. Phys.* **60** 1699–704
Kirkpatrick S 1973 *Rev. Mod. Phys.* **45** 574–88
Pohoryles B and Figielski T 1975 *Phys. Status Solidi* a **32** 387–93
Sinkkonen J 1981 *Phys. Status Solidi* b **103** 231–7
Springett B E 1973 *Phys. Rev. Lett.* **31** 1463–5
Stroud D I 1975 *Phys. Rev.* B **12** 3368–73
Webman I, Jorther J and Cohen M H 1977a *Phys. Rev.* B **15** 1936–40
—— 1977b *Phys. Rev.* B **15** 5712–23